Nonlinear Electric Circuit Analysis from a Differential Geometric Point of View
Computing Operation Points for a Special Class of Electronic Circuits

The behavior of electrical circuits can be described by a set of algebraic and differential equations (DAE) which can be solved by numerical analysis methods. In this project geometric methods will be used to find operation points.

Especially problematic for the numerical analysis of electronic circuits are non-linear electronic devices, whose functionality is based on the feedback principle or electronic devices, whose voltage/current characteristic includes a region of negative slope (negative differential resistance).

For modeling this class of electronic circuits, it will be necessary to use differential equations with singularities.

Van-Der-Pol-Oscillator

The degenerated Van-der-Pol-Oscillator is a simple circuit consisting of a resistor and a capacitor that are connected in a circle, described by:

\[
\frac{dv}{dt} = -v - v^3 + i \\
\frac{di}{dt} = -9 \cdot v - i^3 + i
\]

with \(v\)-voltage, \(i\)-current and where the differential equation (1) characterizes the capacity and the non-linear relation (2) defines the resistance.

\[
\frac{dv}{dt} = -v - v^3 + i \\
\frac{di}{dt} = -9 \cdot v - i^3 + i
\]

First, the points can be mapped by orthogonal projection on a reflecting perpendicular curve. On the other hand, the points can be mapped by extending the current tangent vector at the time of the bounce. The last option is certainly more difficult to calculate but is still possible in an acceptable time.

Using Homotopy Methods for Finding Starting Points

The methods described above assume that a starting point can easily be found. This is not true for manifolds with a co-dimension greater than one. To acquire such a point it is possible to use homotopy methods which have become a powerful tool in finding solutions of various nonlinear problems, such as zeros or fixed points of maps.

A distinctive advantage of the homotopy method is that the algorithm generated by it exhibits the global convergence under weaker conditions. The homotopy concept is used to determine solutions of high-dimensional nonlinear equilibrium equations by initially solving a simpler problem and then systematically transforming it to the actual problem by embedding it in a homotopy.

Consider the non-linear equation \(g(x) = 0\), with \(x\) being a regular value. The implicit function theorem implies the existence of a curve \(x(\lambda)\) which solves:

\[
H(x, \lambda, x(\lambda)) = g(x(\lambda)) + (1 - \lambda)g(x(n)), \quad \lambda \in [0, 1]
\]

If we formulate the map \(g(x, \lambda)\) so that its zero set is a point on the manifold, the curve \(x(\lambda)\) will converge towards it under certain conditions. This method was used by Moll and Wolter to find solutions for similar geometrical problems. By using these experience this methods can be applied to higher dimensions.

Basic Principles of Tracing Curves on Two-Dimensional Manifolds Embedded in Three-Dimensional Space

The determination of operation points of the previously described system is still not generally solved. There exist homotopy methods that can be used to calculate the isolated zeros of the system of equations. However, these can not easily be used for oscillating systems. Moreover, for example the equations generally used in SPICE simulators are not suitable for the use of homotopy methods.

Our approach consists in the geometric interpretation of the system. We do not want to determine directly the operating points using homotopy methods. Instead, we use the homotopy-methods only to search individual starting points on the critical manifold. From there, numerical algorithms of the differential geometry are used to trace the flow on the manifold.

We can trace a curve on the manifold by numerically integrating the given differential equations which describes a tangent vector field on the manifold. This should lead us to an operation point or, if an oscillating circuit is given, represents the set of states.

Following the curve, we want to consider what happens, when the curve reaches a fold, i.e. a generalized extremum situation on the manifold. This will be the case if the circuit oscillates: the operation point jumps from the extremum to another (non-neighboring) point of the manifold. Therefore we're interested in a submanifold \(S\) of maximum points, as they characterize points where a jump may start. Additionally, we want to determine from the submanifold \(S\) a second one by orthogonal projection \(\pi\) that represents the set of points where a jump can end.

To trace a path on a two-dimensional manifold \(\Sigma\) embedded in the \(2\mathbb{R}^3\) we can define a differentiable manifold by

\[
g(x, y, z) = 0
\]

to define a path on that manifold we use the parameter \(t\):

\[
g(x(t), y(t), z(t)) = 0
\]

Supposing we need the set of maximum points in \(z\)-direction, we differentiate w.r.t. \(z\) by

\[
g(z, x, y) = 0
\]

Differentiate w.r.t. \(z\) leads to:

\[
\frac{d}{dt}g(x(t), y(t), z(t)) = 0
\]

We can easily trace the curve of maxima if we use a function \(f\) of \(x\) and \(y\) instead of \(z\):

\[
g(x(t), y(t), f(x(t), y(t), z(t)) = 0
\]

\[
g(x(t), y(t), f(x(t), y(t), z(t)) = 0
\]

\[
\Rightarrow \frac{df}{dt} = 0
\]

With it we are able to calculate \(f\).

As an example we see in the last Figures the implicit function:

\[
x = f = y = z
\]

The shape of the manifold can be taken as an example set of operation points.

References


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Leibniz Universität Hannover, Institute for Man-Machine-Communication, Welfengarten 1, 30167 Hannover

Dipl.-Math. Philipp Blanke
Dipl.-Ing. Martin Gutsche
Prof. Dr.-Ing. Wolfgang Mathis
Prof. Dr. Franz-Erich Walter