Fast Inverse Forging Simulation via Medial Axis Transform
A geometric approach towards backwards forming simulation.

Objective
In hot-metal drop-forging, the quality of the final product is highly dependent on the design of the forging dies and the layout of the process. Computer-aided techniques are used to reduce design time and to decrease the number of iterations until the final layout is reached. To that end, simulations are developed from equations of general plasticity theory, then Finite Element Analysis (FEA) is employed to find a solution to the so-called partial differential equations. FEA is used to verify the die designs that were accomplished by using empirical relationships or based on engineering practice. For complex parts, several steps are needed to deform the initial simple shape to the end shape with optimal properties and within a geometrical tolerance. Conventional simulation starts with the non-deformed part and results in the final shape, while the engineer has to find relationships between the final product and wants to derive the parameters. These procedures are obviously opposite. We propose a geometric approach for a direct inverse simulation, based on the work of Mathieu et al. [2] which is applied to help the engineer in laying out the process.

An alternative inverse simulation
As an alternative to FEM simulations, Mathieu et al. proposed an approach based on experimental observation and elementary plasticity theory [2]. In drop forging experiments, Mathieu noticed that the material flow followed specific paths. These paths are an application of the Medial Axis concept.

Cut through forging die with velocity field and Medial Axis of the die cavity with two maxima balls.

Definition 1 (Medial Axis Transformations), Let $B$ be a solid in $\mathbb{R}^n$. The Medial Axis $M_A$ of $B$ is defined as the closure of centers of maximal $n$-balls in $B$. The radius function $r : M_A \to \mathbb{R}$ assigns the radius of the corresponding maximal ball to every point in $M_A$.

Definition 2 (Maximal Ball), An $n$-dimensional ball $B_r(c) \subset \mathbb{R}^n$ with center $c \in \mathbb{R}^n$ and radius $r \in \mathbb{R}^n$ is called maximal in $\mathbb{R}^n$ if there exists no other ball $B_{r'}(c) \subset \mathbb{R}^n$ that contains $B_r(c)$.

2D-Algorithm
Input to the algorithm is the geometry of the tool, which is given by a point sample of a planar axis cut through the die surface. We assume that the material exhibits rigid-perfectly plastic behavior. We use a standard Cartesian coordinate system with orthogonal axes $x$, $y$, $z$. The deformed volume has a constant thickness in a direction that is cancelled out in the equations, and the cell will be in the $x$, $y$ plane. Furthermore, we describe the material by its yield curve together with a constant working temperature $T$ and a constant friction coefficient $\mu$. Finally, we require the maximum speed of deformation $\dot{\varepsilon}$ during the process.

The first step is the approximation of the Medial Axis via the Voronoi diagram of the point set, that produces a connected tree of Voronoi edges and vertices. The die area is partitioned, so that each edge $e$ of the tree corresponds to a cell $C$.

Then, we determine the points where the border of the material, i.e., the material front, cuts the medial axis. At these points, which we call end points, material will be removed to fill the volume which will be freed when the dies move. The volume movement depends on resistance along the displacement paths; therefore, we compute a forming resistance for each cell of the partition, based on the following consideration.

Since we are only interested in the final shape of the deformed material and neglect the influence of temperature and deformation history on the process, it is sufficient to look at the movement of the object's boundary.

Forming Resistance
To compute the forming resistance of the cell $C$, corresponding to the edge $E := c_1 - c_2$ between vertices $c_1$ and $c_2$, with footpoints $f_1$ and $f_2$ in distance and $f_1$ and $n$. We substitute it by a simple element as shown in figure 3. We define the height of the substituted element to be $h := r + f_1$, so that $H$ has the height of the element of the partition, based on the following consideration.

With this result from [1], we can compute the forming resistance of each cell.

Resistance along displacement paths
Since we assume that material will be transported along the MA, we still have to determine the volume ratios that are moved along the different branches. We postulate that the material will always move along the path of least resistance. Therefore, the total deformation resistance is added up along each branch of the MA and the displaced volume is distributed accordingly. To avoid multiple summation, the tree structure of the MA can be used to implement a backtracking algorithm, allowing fast computation of resistances for every vertex.

Finally, the distribution of material volume will be calculated iteratively over the cells, depending on the determined resistances and a prescribed rate of transport, e.g., 5% of the volume per time-step. The latter rate is an heuristic, where further research could probably provide more transparent parameters.

Outlook
The 2D algorithm can be lifted to 3D. Approximation of the MA by filtered Voronoi-diagrams is a robust and fast method for the geometric computation that is working well in 3D. The computation can be sped up by detecting the parts of the MA that don’t have to be recalculated. There are obviously the points which have footpoints on the same die part and whose associated medial ball does not intersect other die parts. The geometric forming resistance also has a counterpart in 3D based on cells of the Voronoi diagram, so that a graph-based approach like in 2D will be implemented.

References

This project is a collaboration with Prof. B-A. Binnery and Dipl.-Ing. Hans Cofert, Institut für Umformtechnik at the Leibniz University of Hannover; it is supported with a research scholarship from the Gruaule College 616 of the DPS.