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Structure of Doctoral-Research at the Welfenlab

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RELATION OF CONCEPTS (AROUND 2003)
0. PREFACE: DYNAMICAL SYSTEMS
Dynamical Systems

General form: e.g.

\[ x''(t) = M^{-1} \cdot F(x(t), x'(t)) \]

e.g.: used in Haptex simulations cf. slide 23
DYNAMICAL SYSTEMS

General form: e.g.

\[ x''(t) = M^{-1} \cdot F(x(t), x'(t)) \]

Example: Pendulum

- Energy (area) should be conserved
- Result quality depends on chosen integration method
1. **Computational Geometry and Topology**
Dynamical Systems on Submanifolds in Euclidean Space

Problem Statement

• Study Solutions (Flow Map) \( \varphi(x,t) \) of \( y' = G(x,t) \)
  \[ t \mapsto \varphi(x,t) \quad \text{with} \quad \frac{d}{dt} \varphi(x,t) = G(\varphi(x,t),t) \]

• Analyze dynamical system with orbits on submanifold \( S \)

• \( S \) defined implicitly by \( F(x) = c \)

• Following a point controlled by this system

• Point moves slow while on \( S \) and fast while not on \( S \).
DYNAMICAL SYSTEMS ON SUBMANIFOLDS IN EUCLIDEAN SPACE

Applications

• Analyzing Physical Systems,
  – electrical circuit system
  – Mechanical and biological systems
• Helpful studying periodic orbits

**Special Plastic Deformations & Medial Axis**

**Problem Statement**

- Compute the inverse $\varphi^{-1}$ of a computable Flow map $\varphi(x, t)$
- Interesting in Metal Forming process

- This problem turns out to be very difficult, see:
  - “Current theoretical approaches to collective behavior of dislocations”, G. Ananthakrishna

**SPECIAL PLASTIC DEFORMATIONS & MEDIAL AXIS**

**Problem Statement**

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GEODESICS IN RIEMANNIAN MANIFOLDS

- Described by special dynamical systems defined by the geometry of the manifold
- Differential equation: \( y' = \Gamma(y, t) \)
- Special flow maps: Offset/Exponential maps \( O(y, t) \) offset geometries
- More specialized notation:
  \[ \exp(x, t), \text{inverse: } \exp^{-1}(\exp(x, t)) \]

- Study Focal Sets, i.e. where \( D_{\exp} \) has not maximal rank
- Study Flow Map, its inverse and singularities
  - Use it to analyze the distance geometry of a Riemannian Manifold

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Physical interpretation

• Geodesic $g(t)$ from $p$ to $q$ is local minimizer (stationary point) of energy $\int |g'(t)^2|$ by comparison with close neighbor paths joining $p$ and $q$.
LAPLACE

• Use computations in the context of Laplace operator or its generalizations and modifications to analyze and classify geometry of
  – shapes
  – images

Reason:
Laplace operator and its eigenfunctions and eigenvalues are related to shape of geometric objects.
LAPLACE

Wave equation: \[ u_{tt}(t, x) = \Delta_x u(t, x) \]
Heat equation: \[ u_t(t, x) = \Delta_x u(t, x) \]
Separation of variables: \[ u(t, x) = \nu(x) \cdot T(t) \]
Eigenvalue equation: \[ \Delta \nu = \lambda \nu \]

Eigenvalues (spectrum): \[ \lambda_1, \lambda_2, \ldots \]
Eigenfunctions: \[ \nu_1, \nu_2, \ldots \]

Heat Kernel: \[ K(t, x, y) = \sum_{i=1}^{\infty} e^{-\lambda_i t} \nu_i(x) \nu_i(y) \]
Heat Trace: \[ \int_M K(t, x, y) = \sum_{i=1}^{\infty} e^{-\lambda_i t} \xrightarrow{t \to 0} A_S(c_0, c_1, c_2, t) \]
LAPLACE SPECTRA AS SHAPE-DNA

• Theorem: Isometric shapes => Same Laplace spectra


SHAPE-DNA FOR GEOMETRY

Applicable for:
• 2D surfaces
• 3D volumetric data

SHAPE-DNA FOR IMAGES

Applicable for:
• Monochromatic images
• Greyvalue images
• RGB images
• 3D (Voxel) images

LAPLACE: SPECTRAL INFORMATION

Eigenvalues/Heat Trace determine:
• Volume
• Volume of boundary
• Curvature integrals
• Euler characteristic
• ...

Spectrum can be used as „Shape-DNA“ for
• Surfaces
• Solids
• Images
• ...

GENERALIZATIONS AND CURRENT RESEARCH

• Laplacians on vector bundles
  – Zero-sets of eigenfunctions
  – Hyperbolic manifolds


• Extended descriptors (partial matching, ...)

2. VIRTUAL REALITY
VIRTUAL REALITY: BASICS

Three important areas for realistic Virtual Worlds

A. Visual
B. Tactile
C. Haptic
Physical Simulation  Tactile Signal

- Physical Models yields relevant Tactile Data $T(t)$
- Signal (usually) created by a Tactile array
- Relevant class of Signals to create illusion (Tactile Color)


HAPTIC SIMULATION

- Solve a Dynamical System
  \[ M \cdot x''(t) = F(x(t), x'(t)) \]
  \[ x''(t) = M^{-1} \cdot F(x(t), x'(t)) \]
- Appropriate Integration needed
- Usually fast methods are required to achieve 1000Hz

HAPTIC SIMULATION

- Interpretation as a particle system
- Second order ODE:
  \[ r''(t) = F(r, r')M^{-1} \]
- Numerical integration of a \(6n \times 6n\) matrix
3. Physical Simulation
GENERAL EXTENDABLE PHYSICAL SIMULATION FRAMEWORK

UPSIDE

Universal Parallel Simulations and Distributed Environments
Computational Methods for Fluid Dynamics

• Examination of different computational paradigms
  – Grid based (esp. Adaptive multigrid solvers)
  – Particle based (SPH, WCSPH, PCISPH, ISPH...)

• Research on fluid structure interaction (focusing on haptics)

• Relations to Riemannian Geometry as in “Geometrical theory of fluid flows and dynamical systems”, Tsutomu Kambe

HEARING MECHANICS

2D Simulation

- Acoustic streaming analysis
- FEM solution for cochlea mechanics in every single time step (involving Navier-Stokes, etc.)
- Solving a huge dynamical system

\[ M(t) \frac{\partial^2 u}{\partial t^2}(t) + H(t) \frac{\partial u}{\partial t}(t) + K(t) u(t) = F(t) \]

Hearing Mechanics

3D Simulation
- Extension to the 3D case
- Non-FEM approach (FDM / Lattice Boltzman)
- Use Cluster for calculations
HAPTIC INTERACTION WITH 3D VOXEL DATA

- Haptic Realtime Challenge
- Similar to “normal Haptic Simulation”
- Use raycasting to check for contacts
- Allow interactive deformations with haptic feedback
- Integrated into the YaDiV framework


FLIGHT PATH SIMULATION

Problem statement:
Many simultaneous flights

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Problem statement:
Many simultaneous flights

Get a collision free dynamical system
  – Use a 4 dimensional description (Hexadecimal Tree)
  – Search adaptively in the 4 dimensional space
Problem statement:
Many simultaneous flights

Get a collision free dynamical system

Consider delaying flights to avoid collisions

The system has to be very fast to be of use
4. VISUALIZATION
**Visualization based on Fractal Models**

- Discrete dynamic system generated by a self similar map $\lim(S^n(I))$
  - with $I$ being a initial seed set.
- Limit set $\rightarrow$ fractal
- Used to generate visualization data

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3D Voxel Based-Volume Visualization

- A function $P(x)$ defined on 3D Voxel data
- Visualization of intensity data of this function (YaDiV)

• Some of the 3D biomedical data can be visualized with YaDiV
• Important to handle processes that change over time on different time and length scales
• Visualized data
  – Computational results
  – Appropriate physical models involved

Example
• Biomedical CT-Data
  – Radon transform density function
  – Segmentation visualizable data

Computational Geometry + Topology

Visualizing Data, Processes, Calculations

Physical Simulation

Simulation + Geometric Analysis of physical systems

Flows, Dyn. Syst. on Manifolds

Simulation + Geometric Analysis of physical systems

Metal Forming, Plastical Deformation

Physical interpretation of energetic minima

Physical interpretation of electrodynamic Systems, etc.

Application: Shape and Image cognition

Uniscalar modelling, Fractals

Biomedical Multiscale Visualization

Modeling Planets combined with Fluid Models

Shape and Image cognition

Flight Path Simulation

Fluid mechanics

Time- and space adaptive integration

Generalized Physical Framework

Physical interpretation of energetic minima

Physical interpretation of electrodynamic Systems, etc.

Laplace and generalizations

Eye Tracker

Tactile Illusion

Perceptive Illusion (Haptics)

Haptics

Virtual Reality